Investigating Quantum Chromo Dynamics (QCD) on the Lattice

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1. Introduction, motivation

2. Hadron spectroscopy, the role of QCD

3. Cluster computing for lattice QCD

4. Recent excitement: QCD at nonvanishing densities

5. Conclusions

• QCD is a generalized, extended version of QED

in QED only one charge: electric (positive or negative)
in QCD three charges: we call it colour (red, blue, green)
all of them can be positive or negative (not real colours)

gluons (similarly to photons) transmit the strong interaction
between quarks (which are similar to electrons)

neither free gluons nor quarks were observed experimentally

in QED: photons are described by A, electrons by ψ

$$-\frac{1}{4g^2}F_{\mu\nu}F^{\mu\nu}+\bar{\psi}[i\gamma_\mu(\partial^\mu+iA_\mu)+m]\psi,\ F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$$

in QCD: field A is a traceless 3*3 matrix, ψ has and index

$$-\frac{1}{4g^2}\mathrm{tr}F_{\mu\nu}F^{\mu\nu}+\bar{\psi}\{i\gamma_\mu(\partial^\mu+iA_\mu)+m\}\psi,\ F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu+i[A_\mu,A_\nu]$$

gauge invariance unambiguously fixes this form

• Our understanding of particle physics:

QUANTUM FIELD THEORY

field variables (e.g. $\vec{E}(\vec{r},t)$) are treated as operators

• Interactions

symmetries + internal consistency

$\Rightarrow$ gives the interactions unambiguously

GREAT SUCCESS

precision perturbative predicitions: magnetic moment of e^-

upto 13 digits complete agreement with experiments

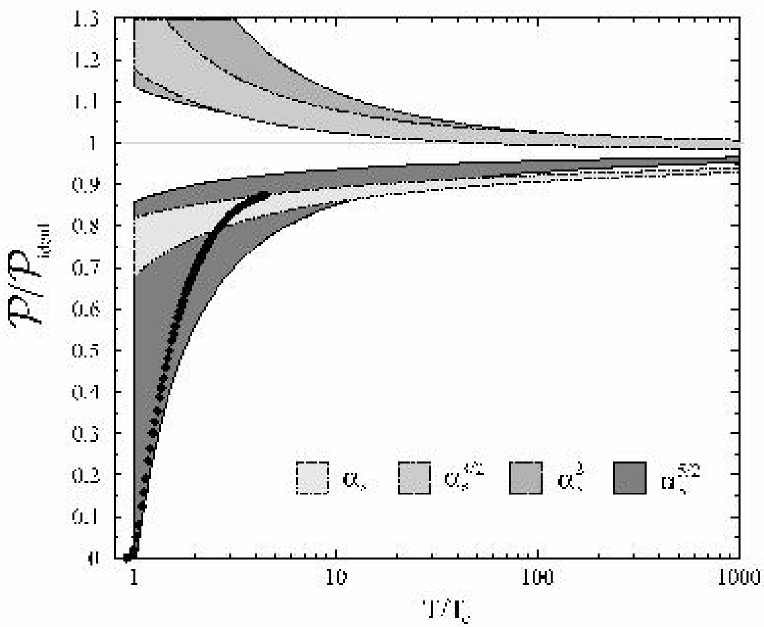
$\mu_e = 2.002319304374(8)$, theory

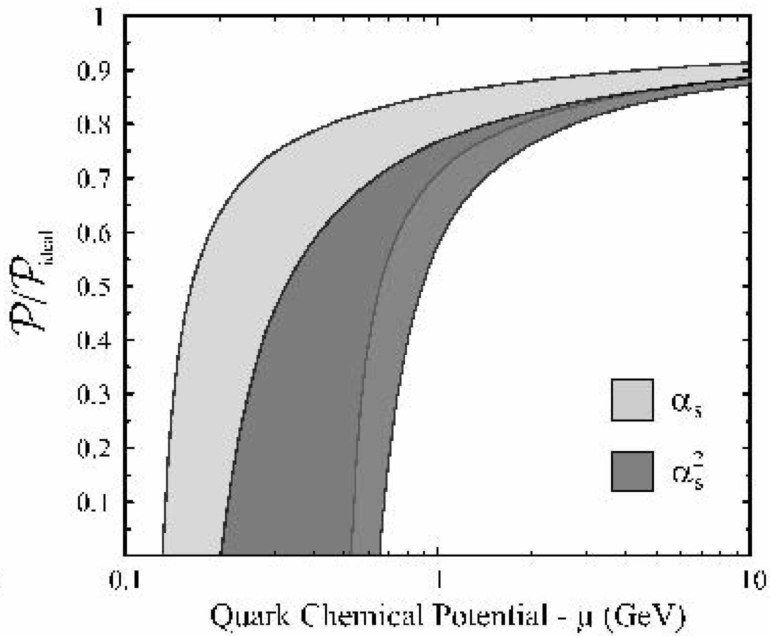
$\mu_e = 2.002319304402(27)$, experiment

• in other cases extremely bad convergence:

pressure at high T or μ (tunes baryon density)

converges at $T=10^{300}$ MeV and $\rho=1000\cdot\rho_0$





• even worse: no sign of the same physical content

Lagrangian contains quarks and gluons

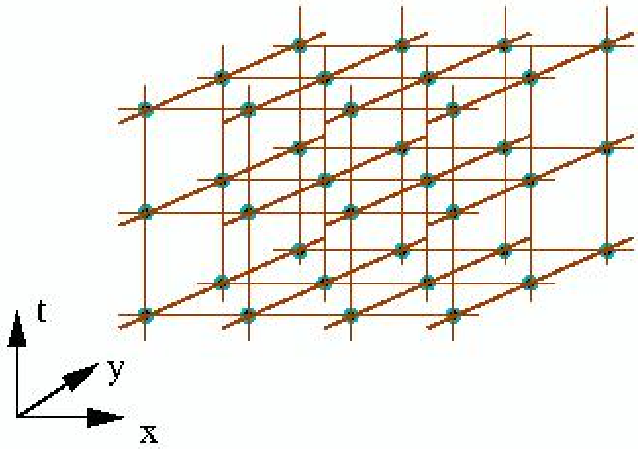
we detect none of them, they are confined

we detect instead composite particles: protons, pions

- other systematic approach:

QUANTUM FIELDS ON THE LATTICE

quantum theory: path integral formulation
quantum mechanics: for all possible paths add exp(iS)
quantum fields: for all possible field configurations add exp(iS)
Euklidean space-time ($t=i\tau$): exp(-S) sum of Boltzmann factors
formally: four-dimensional statistical system



fine lattice $\Rightarrow$ resolve the structure of the proton
10^9 dimensional integrals $\approx$ 100–500 Gflops is needed

Hadron spectroscopy in lattice QCD

Determine the transition amplitude between:
having a "particle" at time 0 and the same "particle" at time t
$\Rightarrow$ Euclidean correlation function of a composite operator $\mathcal{O}$:

$$\langle 0|\mathcal{O}(t)\mathcal{O}^\dagger(0)|0\rangle$$

insert a complete set of eigenvectors $|i\rangle$

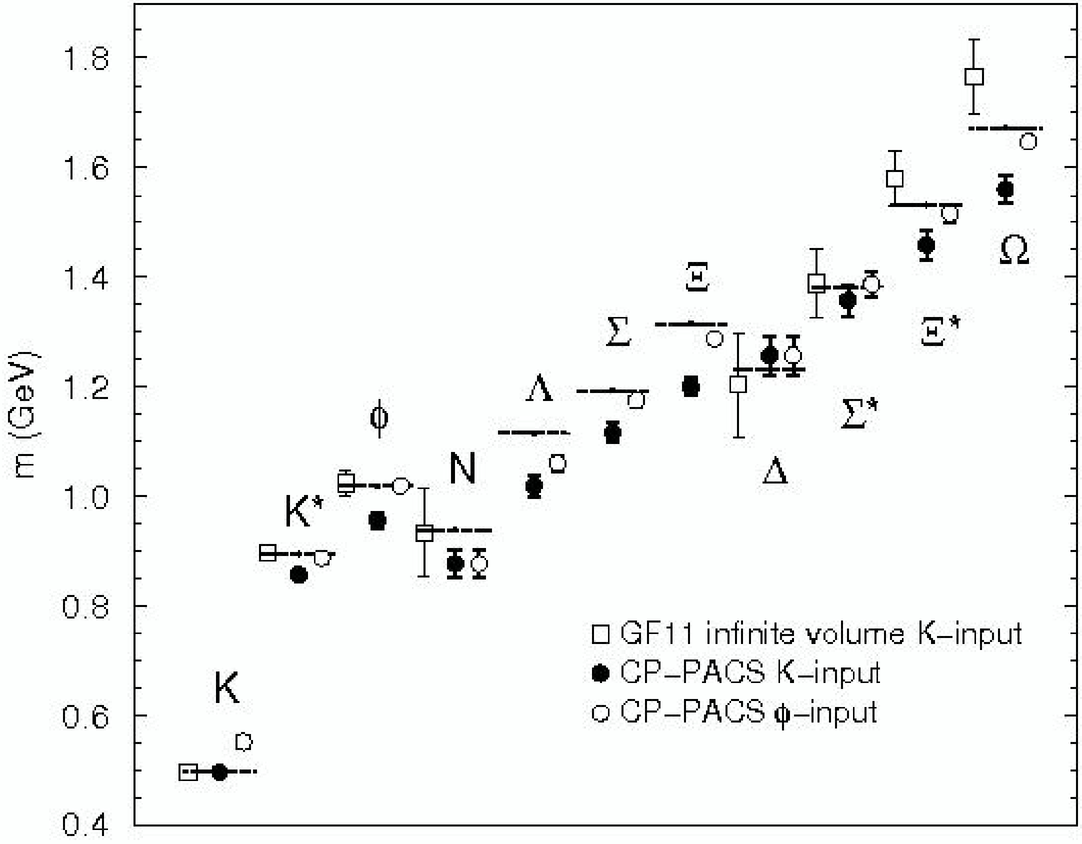
$$= \sum_i \langle 0|\mathrm{e}^{-Ht}\ \mathcal{O}(0)\ \mathrm{e}^{Ht}|i\rangle\langle i|\mathcal{O}^\dagger(0)|0\rangle = \sum_i |\langle 0|\mathcal{O}^\dagger(0)|i\rangle|^2\ \mathrm{e}^{-(E_i-E_0)t},$$

where $|i\rangle$: eigenvectors of the Hamiltonian with eigenvalue E_i.

and $$\mathcal{O}(t) = \mathrm{e}^{-Ht}\ \mathcal{O}(0)\ \mathrm{e}^{Ht}.$$

t large $\Rightarrow$ Lightest states (created by $\mathcal{O}$) dominate.
$\Rightarrow$ Exponential fits give E_i's

hadronic spectrum: no more no less than the detected one



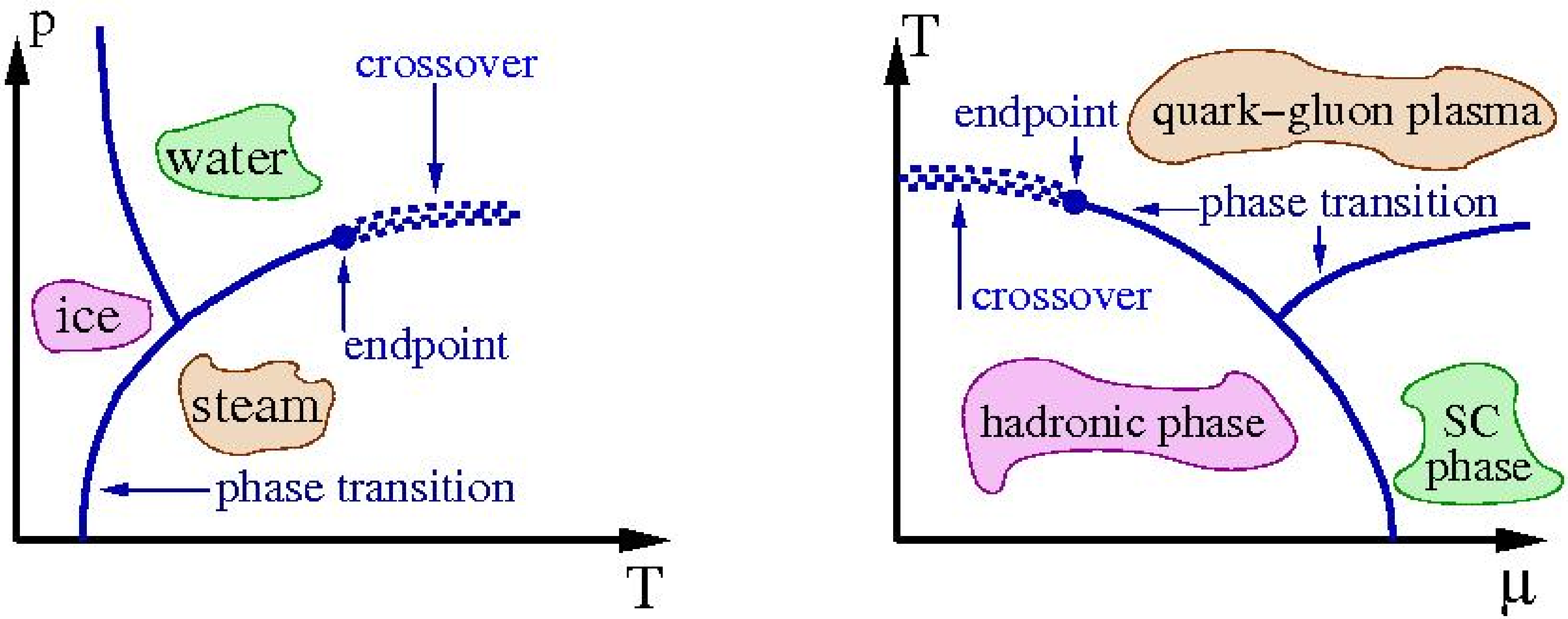
lattice results show confinement

no sign of free quarks or gluons

- unfortunatley Monte-Carlo techniques work only in vacuum, since 20 years no results at finite density (chemical potential)

• Strong interaction: confines quarks into protons

high T or large density (large momenta) deconfining transition



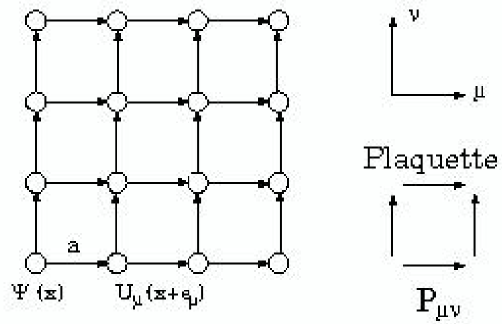
till 2001 „ab initio" result only for μ=0 (chemical potential)

one of the most difficult problems of lattice QDC

reason: sign problem at nonvanishing density (solid state physics)

locate the endpoint by an „ab initio" technique

lattice action of QCD and Monte-Carlo techniques



$$\mathscr{L} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + \bar{\psi}(D_\mu \gamma^\mu + m)\psi$$

anti-commuting $\psi(x)$ quark fields live on the sites

gluon fields, $A^a_\mu(x)$ are used as links and plaquettes

$$U(x,y) = \exp(ig_s \int_x^y dx'^\mu A^a_\mu(x') \lambda_a/2)$$

$$P_{\mu\nu}(n) = U_\mu(n) U_\nu(n+e_\mu) U^\dagger_\mu(n+e_\nu) U^\dagger_\nu(n)$$

$S = S_g + S_f$ consists of the pure gluonic and the fermionic parts

$$S_g = 6/g_s^2 \cdot \Sigma_{n,\mu,\nu} \left[1 - \mathrm{Re}(P_{\mu\nu}(n))\right]$$

quark differencing scheme:

$$\bar\psi(x)\gamma^\mu\partial_\mu\psi(x) \to \bar\psi_n\gamma^\mu(\psi_{n+e_\mu} - \psi_{n-e_\mu})$$
$$\bar\psi(x)\gamma^\mu D_\mu\psi(x) \to \bar\psi_n\gamma^\mu U_\mu(n)\psi_{n+e_\mu} + \ldots$$

in continuum the chemical potential acts: $\mu a\bar\psi_x\gamma_4\psi_x$
fourth component of an imaginary(!), constant vector potential

fermionic part as a bilinear expression: $S_f = \bar\psi_n M_{nm}\psi_m$

Euclidean partition function gives Boltzman weights

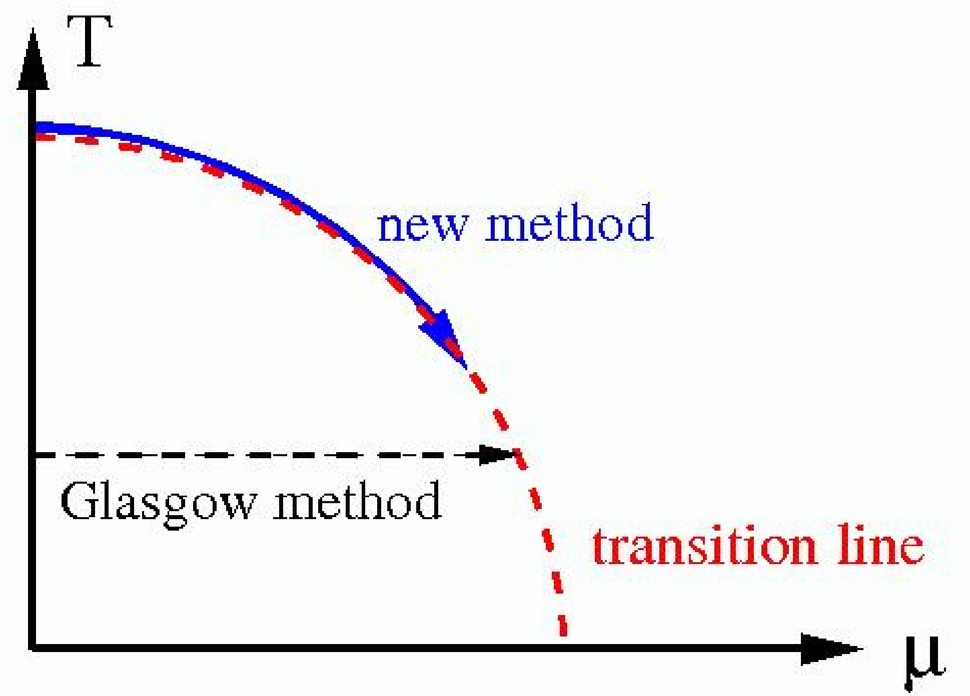
$$Z = \int\prod_{n,\mu}[dU_\mu(x)][d\bar\psi_n][d\psi_n]e^{-S_g-S_f} = \int\prod_{n,\mu}[dU_\mu(n)]e^{-S_g}\det(M[U])$$

Metropolis step for importance sampling:

$$P(U \to U') = \min\left[1, \exp(-\Delta S_g)\det(M[U'])/\det(M[U])\right]$$

for μ=0 the determinant is positive, for $\mu \neq$0 it is complex
$\Rightarrow$ no probability interpretation, no Monte-Carlo method

Comparison with the Glasgow method



one parameter reweighting

single parameter (μ)

purely hadronic configurations

New method

two parameters (μ and β)

transition configurations

- PC-cluster development: local field theory
  optimal price/performance, uses the PC-market

Computer Physics Communication 134 (2001) 139, [hep-lat/9912059]

standard PC-s in a 3 dimensional mesh (periodic)

next neighbours reached by a self-made communication card

lesson: PC industry is faster, we have to use them

Computer Physics Communication 152 (2003) 121, [hep-lat/0202030]

1. driving force: video-games $\Rightarrow$ rotation SO(3) group

locally isomorf to SU(2) group $\Rightarrow$ action can be calculated

2. internet music/video download: gigabit ethernet (switch)

cross twisted cables: connect 2 PC-s; 4 cards: 2 dim. mesh

Phase line and endpoint of 2+1 flavor QCD on $L_t = 4$ lattices

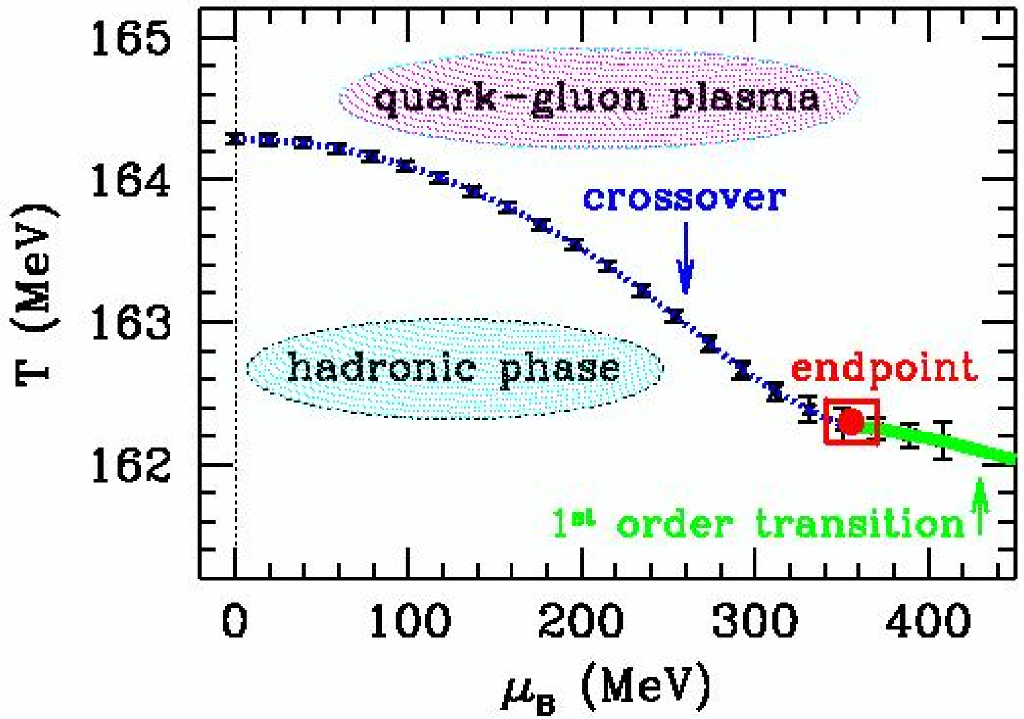
Z. Fodor, S. D. Katz, JHEP 03 (2002) 014, JHEP 04 (2004) 050

- three basic steps of the analysis (m_s physical, m_{ud} physical)

a. determine the transition points β_c as a function of μ

b. inspect V$\rightarrow\infty$ limit to separate crossover and phase transition

c. transform lattice units into physical ones to get (T,μ) plane



endpoint: $T_E = 162 \pm 2$ MeV, $\mu_E = 360 \pm 40$ MeV similar treatment gives also the equation of state

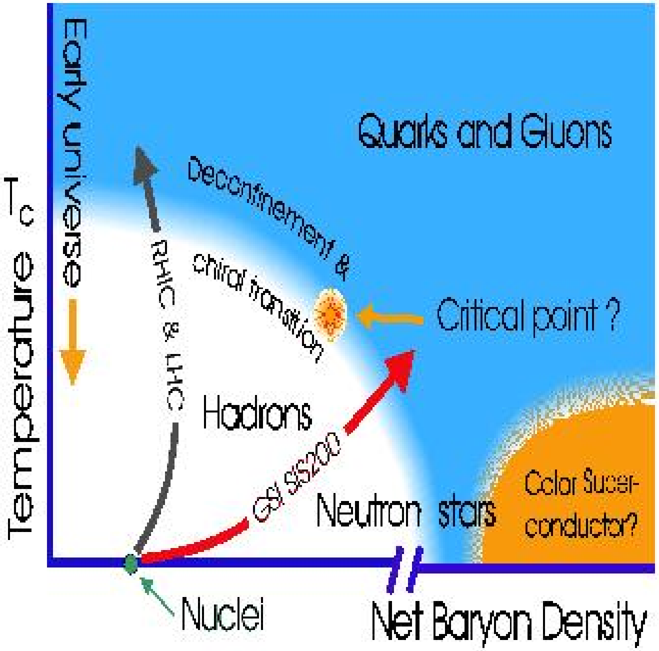
critical endpoint of the phase diagram for water

T=374 °C ρ=0.32 g/cm^3

critical endpoint of the phase diagram for QCD

$T=10^{13}$ °C (160 MeV) $\rho=10^{15}$ g/cm^3 ($5 \cdot \rho_0$)

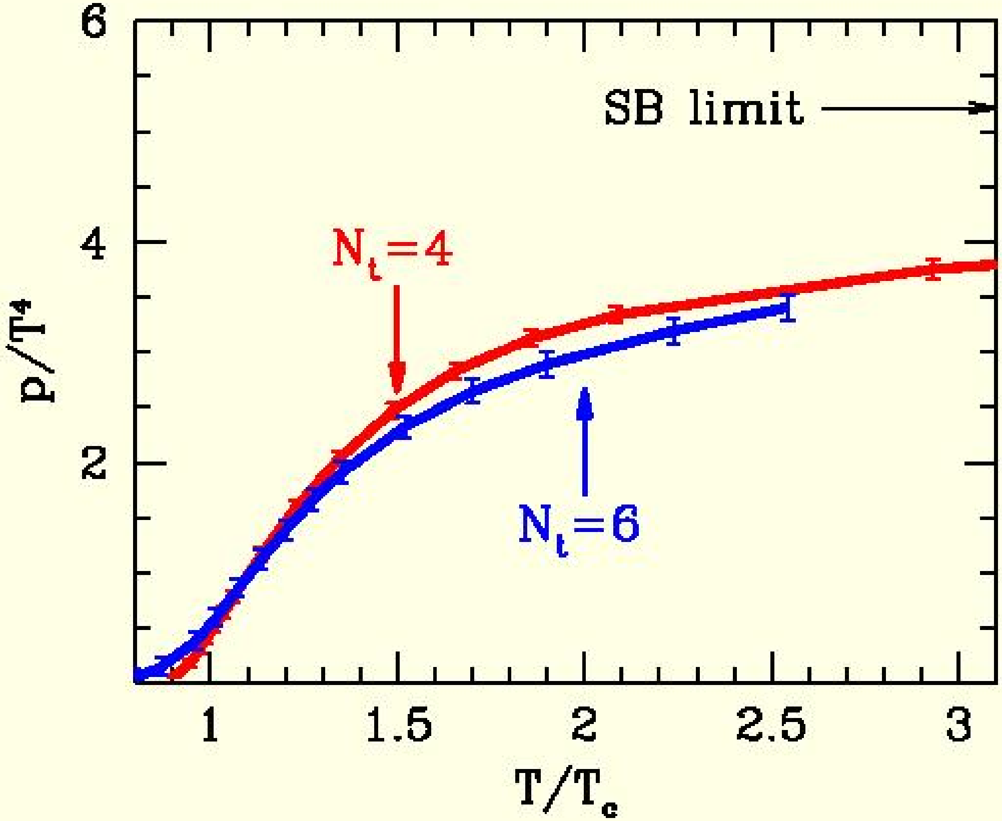
for water a lighter is enough, for QCD extreme conditions



a. big bang: 10^{-6} sec after (ρ is small)
b. neutron-stars (T is small)
c. heavy ion collisions (little bang)

$\Longleftarrow$ GSI/SIS Conceptual Design Report:
www.gsi.de/GSI-Future/cdr

Pressure and scaling



- equation of state for matter(!) at extreme conditions

SUMMARY

- QCD is the complete theory of the strong interactions,
its lattice gauge theory formulation is equivalent
to a four-dimensional classical statistical system

- first principle results for hadronic physics
e.g. spectrum (mass of proton), matrix elements, ...

- QCD at non-vanishing μ suffers from the sign problem,
the same problem as in solid state physics (e.g. Hubbard model)

- recent excitement/results for lattice QCD at $\mu \neq 0$

- new technique: overlap improving multi parameter reweighting

- calculations at the Eotvos University of Budapest
PC cluster: 350 Intel P4 processors, next-neighbour GigE

- results for the critical endpoint and equation of state